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Adoption of Park's Transformation for Inverter Fed Drive

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ABSTRACT

Park's transformation in the context of ac machine is applied to obtain quadrature voltages for the 3-phase balanced voltages. In the case of a inverter fed drive, one can adopt Park's transformation to directly derive the quadrature voltages in terms simplified functions of switching parameters. This is the main result of the paper which can be applied to model based and predictive control of electrical machines. Simulation results are used to compare the new dq voltage modelling response to conventional direct – quadrature (dq) axes modelling response in direct torque control – space vector modulation scheme. The proposed model is compact, decreases the computation complexity and time. The model is useful especially in model based control implemented in real time, in terms of a simplified set of switching parameters.

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1. INTRODUCTION

In three-phase machines usually the behavior and performance are described and analyzed by their voltage and current equations. The coefficients of the differential equations which describes the dynamic behavior of the machines are time varying [1], [2] (except when the rotor is stationary). The mathematical modelling of such a system tends to be complex as the flux linkages, induced voltages, and currents change continuously as the system is in relative motion. For such a complex electrical machine analysis, mathematical transformations [3]-[6] are often used to separate or decouple the variables and to solve equations involving time varying quantities by referring all variables to a common reference frame either stationary or rotating. Among the various methods available for transformation, the well known [8], [9] are: Clarke Transformation and Park Transformation

By proper selection of the reference frame, it is possible to simplify considerably the complexity of the mathematical machine model. While these transformations were initially developed for the analysis and simulation of ac machines, they are now extremely useful tools in the digital control of such machines. As digital control techniques are extended to the control of the currents, torque and flux of such machines, the need for compact, accurate machine models is obvious.

Generally while modelling a drive, the 3-φ voltages V_a, V_b, V_c are generated through a switching model of the inverter. Using Parks transformation, quadrature voltages V_d, V_q are then obtained from V_a, V_b, V_c. In this paper, we present a new approach to obtain V_d, V_q voltages directly in terms of a simplified form of switching parameters of the inverter. This result is made possible by combining the switching equations and the Parks transformation and using some regularity found in the coefficients involved. This methodology which gives V_d, V_q directly in terms of a simplified set of switching parameters will be useful in any modelling of inverter based drive, especially in the context of real time control. In order to verify our model output V_d, V_q using the proposed method, we use the instance of Direct torque control (DTC) of permanent magnet synchronous motor (PMSM) employing space vector modulation (SVM) technique. We consider PMSM because of its advantage over other electrical machines and of its wide applications [10], [11].

The rest of the paper is organized as follows: Section 2 gives a simple introduction to Voltage source inverter. Section 3 describes the conventional d-q voltage modelling of PMSM. Section 4 explains the proposed adoption of Park's transformation to reconstruct d-q voltages directly. In Section 5, the d-q voltages are simulated using Matlab/Simulink and the results obtained are compared with those obtained using the proposed model. Section 6 concludes the paper.

2. SWITCHING STATES OF VOLTAGE SOURCE INVERTER

The power devices of the voltage source inverter are assumed in ideal condition: the voltage across the switch is zero when the switches are conducting and there will be voltage across the switch when it is in open circuit in the blocking mode. Therefore, each inverter leg can be represented as an ideal switch. It gives the possibility to connect the three phase windings of the motor to positive or negative terminals of the dc link (V_{dc}). Thus the equivalent scheme for three-phase inverter and possible eight combinations of the switches in the inverter are shown in Figure 1.

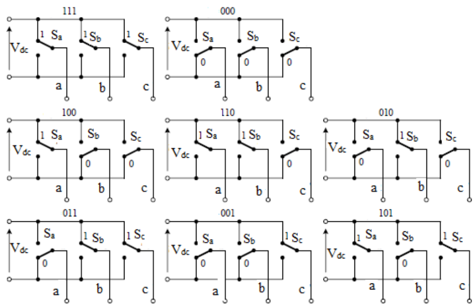

Figure 1. Eight Possible Switching states of Voltage Source Inverter

The relation between the switching states and the inverter voltage outputs in terms of phase and line voltages is given in Table 1.

Table 1. Switching patterns and output vectors

Voltage vectors	Switching vectors			Line to neutral voltage			Line to line voltage		
	S_a	S_b	S_c	V_{an}	V_{bn}	V_{cn}	V_{ab}	V_{bc}	V_{ca}
V_0	0	0	0	0	0	0	0	0	0
V_1	1	0	0	2/3	-1/3	-1/3	1	0	-1
V_2	1	1	0	1/3	1/3	-2/3	0	1	-1
V_3	0	1	0	-1/3	2/3	-1/3	-1	1	0
V_4	0	1	1	-2/3	1/3	1/3	-1	0	1
V_5	0	0	1	-1/3	1/3	2/3	0	-1	1
V_6	1	0	1	1/3	2/3	1/3	1	-1	0
V_7	1	1	1	0	0	0	0	0	0

3. CONVENTIONAL d-q MODELLING

The stator voltage components applied to the electrical machine are estimated using the switching states and dc link voltage (V_{dc}) as follows:

$$V_a = \frac{V_{dc}}{3}(2S_a - S_b - S_c)$$
$$V_b = \frac{V_{dc}}{3}(2S_b - S_a - S_c)$$
$$V_c = \frac{V_{dc}}{3}(2S_c - S_a - S_b) \quad (1)$$

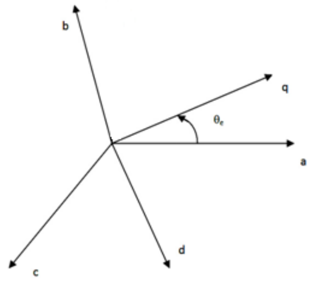

Figure 2. Phasor diagram showing abc and d-q reference frame

Figure 2 represents the phasor diagram of 3-φ rotating machine in dq reference frame. For transforming the three phase voltages into direct-quadrature (d-q) axes voltages, Parks transformation is applied.

Parks transformation of phase voltages is given by:

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \frac{2}{3} \begin{bmatrix} cos\,\theta_e & cos(\theta_e - \frac{2\pi}{3}) & cos(\,\theta_e + \frac{2\pi}{3}) \\ sin\,\theta_e & sin(\theta_e - \frac{2\pi}{3}) & sin(\theta_e + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{2}$$

Where θ_e are the electrical angle of phase a with respect to the reference frame.

4. RECONSTRUCTED d-q VOLTAGES BY ADAPTING PARK'S TRANSFORMATION

The three phase voltages V_a, V_b, V_c which are expressed in terms of switching states in (1) can be put in matrix form as follows,

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \frac{V_{dc}}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} S_a \\ S_b \\ S_c \end{pmatrix} \tag{3}$$

By applying Parks transformation as mentioned in (2) on both sides of the (3) it is possible to transform three phase time varying variable into time invariant variables in terms of quadrature and direct axes as follows,

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} \cos\theta_e & \cos(\theta_e - \frac{2\pi}{3}) & \cos(\,\theta_e + \frac{2\pi}{3}) \\ \sin\theta_e & \sin(\theta_e - \frac{2\pi}{3}) & \sin(\theta_e + \frac{2\pi}{3}) \end{bmatrix} * \frac{V_{dc}}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} S_a \\ S_b \\ S_c \end{pmatrix} \tag{4}$$

The above equation can be simplified to Equation (5) and (6) as below:

$$V_q = \frac{2V_{dc}}{3} \left(cos\,\theta_e S_a + \frac{1}{2}(\sqrt{3}\,sin\,\theta_e - cos\,\theta_e\,)\,S_b - \frac{1}{2}(\sqrt{3} sin\theta_e + cos\,\theta_e\,)\,S_c\right) \tag{5}$$

$$V_d = \frac{2V_{dc}}{3} \left(sin\,\theta_e S_a - \frac{1}{2}(\sqrt{3}\,cos\,\theta_e + sin\,\theta_e\,)\,S_b + \frac{1}{2}(\sqrt{3}\,cos\,\theta_e - sin\,\theta_e\,)S_c\right) \tag{6}$$

Substituting for switching state values of S_a, S_b, S_c, using Equation (5) & (6), the Table 2 is computed as below:

Table 2. Lookup Table for V_d and V_q

Switching States S_a	S_b	S_c	V_q	V_d
0	**0**	**0**	0	0
1	**0**	**0**	$\frac{2V_{dc}}{3}\cos\theta_e$	$\frac{2V_{dc}}{3}\sin\theta_e$
1	**1**	**0**	$\frac{V_{dc}}{3}(\sqrt{3}\ \sin\theta_e+\cos\theta_e)$	$\frac{V_{dc}}{3}(\sin\theta_e-\sqrt{3}\cos\theta_e)$
0	**1**	**0**	$\frac{V_{dc}}{3}(\sqrt{3}\ \sin\theta_e-\cos\theta_e)$	$-\frac{V_{dc}}{3}(\sin\theta_e+\sqrt{3}\cos\theta_e)$
0	**1**	**1**	$-\frac{2V_{dc}}{3}\cos\theta_e$	$-\frac{2V_{dc}}{3}\sin\theta_e$
0	**0**	**1**	$-\frac{V_{dc}}{3}(\sqrt{3}\ \sin\theta_e+\cos\theta_e)$	$\frac{V_{dc}}{3}(\ \sqrt{3}\cos\theta_e-\sin\theta_e)$
1	**0**	**1**	$\frac{V_{dc}}{3}(\cos\theta_e-\sqrt{3}\ \mathrm{sins}\,\theta_e)$	$\frac{V_{dc}}{3}(\sin\theta_e+\sqrt{3}\cos\theta_e)$
1	**1**	**1**	0	0

By inspection of Table 2, we can draw Table 3 by segregating the entries in terms of the orthogonal functions $\cos\theta_e$ and $\sin\theta_e$.

Table 3. V_d and V_q in terms of sine and cosine function under various switching states

Switching States			V_q		V_d	
S_a	S_b	S_c	$\cos\theta_e$	$\sin\theta_e$	$\cos\theta_e$	$\sin\theta_e$
0	**0**	**0**	0	0	0	0
1	**0**	**0**	$\frac{2V_{dc}}{3}\cos\theta_e$	0	0	$\frac{2V_{dc}}{3}\sin\theta_e$
1	**1**	**0**	$\frac{V_{dc}}{3}\cos\theta_e$	$\frac{V_{dc}}{\sqrt{3}}\sin\theta_e$	$-\frac{V_{dc}}{\sqrt{3}}\ \cos\theta_e$	$\frac{V_{dc}}{3}\sin\theta_e$
0	**1**	**0**	$-\frac{V_{dc}}{3}\cos\theta_e$	$\frac{V_{dc}}{\sqrt{3}}\sin\theta_e$	$-\frac{V_{dc}}{\sqrt{3}}\cos\theta_e$	$-\frac{V_{dc}}{3}\sin\theta_e$
0	**1**	**1**	$-\frac{2V_{dc}}{3}\cos\theta_e$	0	0	$-\frac{2V_{dc}}{3}\sin\theta_e$
0	**0**	**1**	$-\frac{V_{dc}}{3}\cos\theta_e$	$-\frac{V_{dc}}{\sqrt{3}}\sin\theta_e$	$\frac{V_{dc}}{\sqrt{3}}(\cos\theta_e$	$-\frac{V_{dc}}{3}\sin\theta_e$
1	**0**	**1**	$\frac{V_{dc}}{3}\cos\theta_e$	$-\frac{V_{dc}}{\sqrt{3}}\sin\theta_e$	$\frac{V_{dc}}{\sqrt{3}}\ \cos\theta_e)$	$\frac{V_{dc}}{3}\sin\theta_e$
1	**1**	**1**	0	0	0	0

Defining V_q, V_d as in (8) below, we have:

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \left[\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}\cos\theta_e + \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}\sin\theta_e\right] \tag{7}$$

Where α_i and β_i i=1,2 are the switching parameters defined in terms of switching states as in Table 4.

We can draw Table 4 by taking data from Table 3 as follows:

Table 4. α and β values for V_d and V_q under various switching states

Switching States			V_q		V_d	
S_a	S_b	S_c	α_1	β_1	α_2	β_2
0	**0**	**0**	0	0	0	0
1	**0**	**0**	$\frac{2}{3}$	0	0	$\frac{2}{3}$
1	**1**	**0**	$\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
0	**1**	**0**	$-\frac{1}{3}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
0	**1**	**1**	$-\frac{2}{3}$	0	0	$-\frac{2}{3}$
0	**0**	**1**	$-\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$
1	**0**	**1**	$\frac{1}{3}$	$\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	$\frac{1}{3}$
1	**1**	**1**	0	0	0	0

Further by inspection of Table 4, we noticed that for all switching states $\alpha_1 = \beta_2$, $\alpha_2 = -\beta_1$. Equation (7) can thus be rewritten as,

$$V_q = V_{dc}(\alpha_1 \cos\theta_e - \alpha_2 \sin\theta_e) \tag{8}$$

$$V_d = V_{dc}(\alpha_2 \cos\theta_e - \alpha_1 \sin\theta_e) \tag{9}$$

Where α_1 and α_2 are as given in Table 5.

Table 5. α_1 and α_2 values under various switching states

Switching States			α_1	α_2
S_a	S_b	S_c		
0	**0**	**0**	0	0
1	**0**	**0**	$2/3$	0
1	**1**	**0**	$1/3$	$1/\sqrt{3}$
0	**1**	**0**	$-1/3$	$1/\sqrt{3}$
0	**1**	**1**	$-2/3$	0
0	**0**	**1**	$-1/3$	$-1/\sqrt{3}$
1	**0**	**1**	$1/3$	$-1/\sqrt{3}$
1	**1**	**1**	0	0

Remark: V_q, V_d given in (8) and (9) identify the quadrature variables in terms of switching states (represented by α_1 and α_2) and the continuous variable θ_e. This is a new result derived from the direct approach used in the paper.

5. SIMULATION RESULTS AND ANALYSIS

In order to compare the proposed model output with that of a inverter output (applied to the machine), simulation involving space vector modulation in a direct torque control scheme is employed. The schematic diagram shown in the Figure 2 was implemented and simulated for permanent magnet synchronous motor in the Matlab-Simulink environment using SimPower System. The input voltage of this PMSM simulation is in terms of V_q,V_d. This d-q voltage is built using Park's transformation which responds according to the switching states S_a, S_b, S_c as per (2). The switching state varies according to the error torque and error flux of the machine. The block within the dashed lines in Figure 2 indicates the algorithm which incorporates the proposed direct approach employing (8) and (9) and Table 5 to evaluate d-q voltages. All the simulations were performed for a 3-φ, 4-pole PMSM motor under no load condition as shown in the Table 6.

Table 6. PMSM parameters

Rated Phase Voltage	300V
Rated Torque	1.7Nm
Magnetic Flux Linkage	0.1848Wb
Pole pairs	2
Rated Speed	3000rpm
Stator Resistance	4.765Ω
Inductance – Lq	0.014H
Inductance – Ld	0.014H

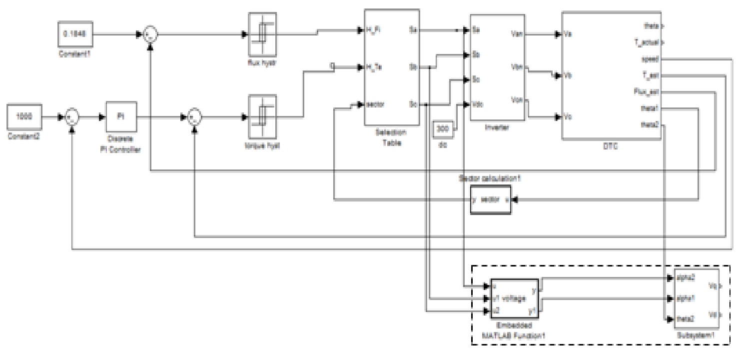

Figure 2. Simulink block diagram of conventional and proposed method

Figure 3 indicates the switching states S_a, S_b, S_c corresponding to flux and torque error from PMSM. Figure 4 indicates the corresponding switching parameters α_1 and α_2 for various switching states S_a, S_b, S_c as per Table 5. Figure 5 represents the comparison of computed direct and quadrature voltages (a& c) which show an almost identical response as measured direct and quadrature voltages (b& d) in the same figure. Figure 6 represents the error in computed and measured voltages of direct and quadrature axes which is of the order of $1.4*10^{-12}$. Figure 7 compares the responses of measured and computed direct and quadrature axes voltages when the input voltage (V_{dc}) is changed to half the rated value at t=0.3s and double the rated value at t=0.62s. The results show that the responses are almost identical for various voltage changes both for the conventional and the proposed techniques.

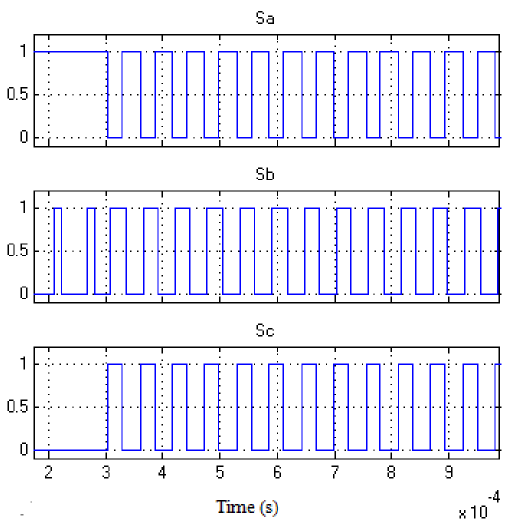

Figure 3. Switching States S_a, S_b, S_c

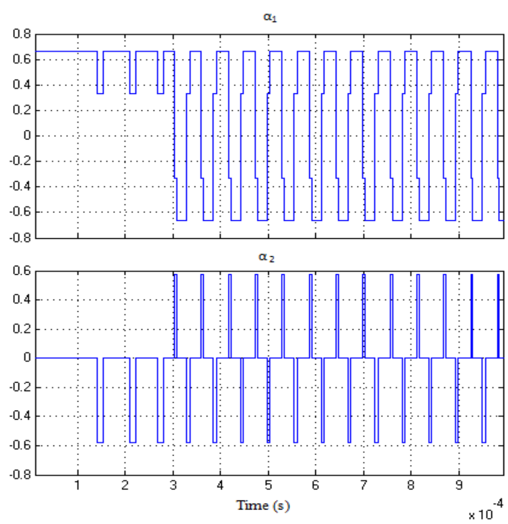

Figure 4. α_1 and α_2 values under various switching states

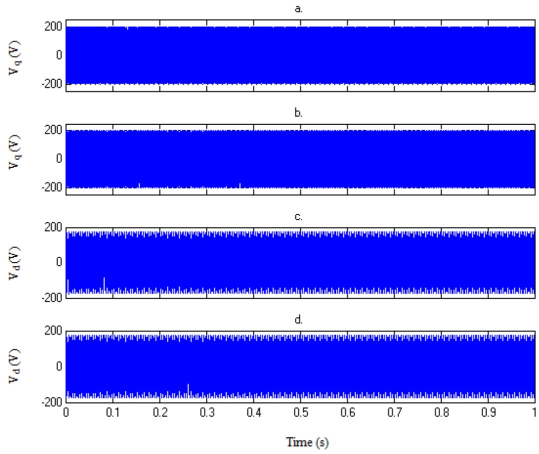

Figure 5. Comparison of computed (a) Quadrature and (c) Direct axes voltages with the measured (b) Quadrature and (d) Direct axes voltages

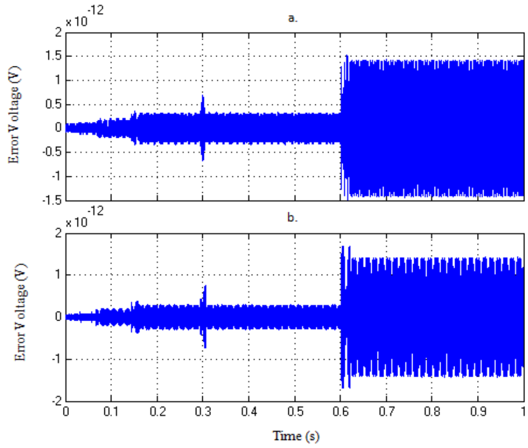

Figure 6. Error Voltage in computed and measured

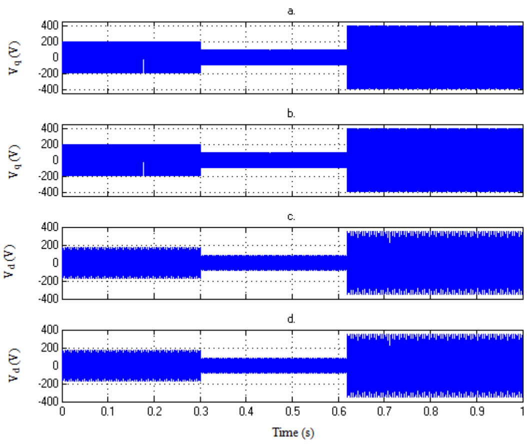

Figure 7. Comparison of computed and measured voltages for various input voltage levels

6. CONCLUSION

This paper presents an adoption of Park's transformation for an inverter fed drive which allows generation of d-q voltages directly in terms of switching parameters. The proposed model has been used in the model based control, such as, indirect torque control and internal model control of PMSM which is our ongoing work.

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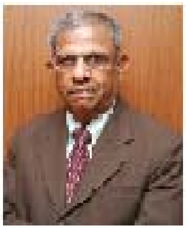

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